SUMMARY

Units

cm/gm/sec (cgs) units are;

dyne = force to accelerate 1 gm at 1 cm/sec^2
erg = work done by 1 dyne cm
unit pole = magnetic pole that exerts 1 dyne on an identical pole 1 cm distant in a vacuum
G = gauss = magnetic field that exerts 1 dyne on a unit pole
maxwell (previously **lines**) = magnetic flux = magnetic flux of field of 1 gauss crossing 1 cm^2
emu of current = current flowing through an arc 1 cm radius, length 1 cm which causes a magnetic field of 1 gauss at the centre of the arc
Gilbert = magneto motive force (**mmf**) = magnetizing force due to an electric current
Oe = **oersted** = magnetizing force per cm length of the magnetic circuit. The symbol for magnetizing force per unit length is **H**. In a vacuum, a magnetizing force of 1 Oe produces a magnetic field of 1 gauss. In other materials, a magnetizing force of 1 Oe produces a magnetic field of μ gauss where μ is the permeability. In air, $\mu = 1$ approximately, in iron μ is over 1000. However in iron, the relation is not linear, μ is not a constant.

Engineering units are;

N = newton = force to accelerate 1 kg at 1 m/sec^2 = 10^5 dynes
J = **joule** = work done by 1 newton metre = 10^7 ergs
W = **watt** = 1 joule/ second = 10^7 ergs/sec
kW = **kilowatt** = 1000 watts
HP =**horse power** = 550 ft lbs/sec = 746 watts
I = **amp** = 1/10 of emu of current
T = **tesla** = magnetic field strength 10^4 gauss. The symbol for magnetic field is **B**
Wb = **weber** = magnetic flux = magnetic flux of magnetic field of 1 tesla crossing 1 m^2. The symbol for magnetic flux is **Φ**. 1 Wb = 10^8 maxwells.
Ampere turns = mmf. A coil N turns carrying a current I amps gives an mmf of N I ampere turns
In a vacuum, a magnetizing force of 1 ampere turn / metre produces a magnetic field of 1.26 10^{-6} tesla.

Basic Formulae $B = \mu H$
Corkscrew rule As a current passes down a wire, the magnetic field follows a circular path the same direction as a corkscrew.

Definition of Volts. The **potential difference** between two points is 1 **volt** if 1 watt of power is dissipated when 1 amp flows from one point to the other. $W = V I$

Ohms Law (for a direct current circuit with resistance R ohms) $V = I R$
Power loss in a resistor $W = I^2 R$
Resistance $R = \rho L (1 + \alpha T) / A$ ohms where ρ is resistivity in ohms per cm cube, L cms is the length, A cm^2 is the cross sectional area, α is temp co-eff and T is the temperature in degrees Celsius.
For Copper $\rho = 1.7 \times 10^{-6}$ ohms per cm cube and $\alpha = 0.004$. At very low temperatures, the resistance of some materials falls to zero
Resistance R_1 in series with R_2. Equivalent resistance = $R_1 + R_2$
Resistance R_1 in parallel with R_2. Equivalent resistance = $1/(1/R_1 + 1/R_2)$

Kirchoff's first law The total current leaving a point on an electrical circuit = total current entering
Kirchoff's second law The sum of the voltages round any circuit = net "I R" drop in the circuit

Force on a conductor in a magnetic field $F = B I L$ Newtons where B in tesla, I in amps and L in metres
Force on parallel conductors $F = [2 I^2 / d]\, 10^{-7}$ Newtons/metre where I is in amps and d is in metres
With currents in opposite directions, the force is pushing the conductors apart

MMF in a solenoid, N turns and current I $mmf = (4\pi / 10) N I$ Gilberts.
Magnetizing Force at the centre of a long solenoid $H = (4\pi / 10) N I / L = 1.26 N I / L$ Oersteds where L is the length in cms and (N I) is the ampere turns
Magnetic field at the centre of a long solenoid length L metres $B = 1.26 \mu N I \; 10^{-6} / L$ tesla.
In magnetic materials, μ is not a constant and the maximum useful value of B is about 1 Tesla
Magnetic flux in a uniform closed magnetic circuit length L metres and cross section A square metres is $\Phi = 1.26 N I \mu A \times 10^{-6} / L$ Wb.
Closed magnetic circuit ie a solenoid wound on a ring or the field circuit of an electrical machine, mmf = sum of mmfs to drive same Φ in each part $\Phi = 1.26 N I \times 10^{-6} / \Sigma(L_1/\mu_1 A_1)$ Where Φ is in weber, I in amps, A in m^2 and L in metres.

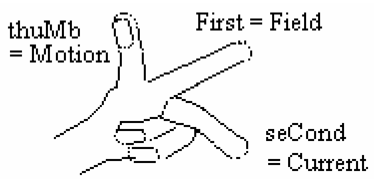
Pull of Electromagnet $Pull = B^2\ 10^7\ / (8\pi)$ newtons per m^2 of magnet face where B is in tesla

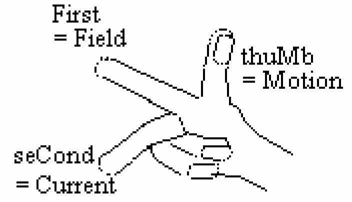
Induced emf $E = -N\ d\Phi/dt$ where E is in volts, N is number of turns and $d\Phi/dt$ is in Wb/sec
This equation is the foundation on which Electrical Engineering is based.

Self Inductance $E = -L\ dI/dt$ where E is in volts, L is inductance in henries and dI/dt is in amps/sec
Self inductance of a coil wound on a ring of permeability μ is $L = 1.26\ N^2\ \mu\ A / S \times 10^{-6}$ Henries where N is number of turns, A is cross sectional area in m^2 and S metres is the length of the magnetic circuit. Experimental results for a coil length S metres, diameter d metres and radial thickness t metres with air core indicate $L = 3\ d^2\ N^2 / (1.2\ d + 3.5\ S + 4\ t)$ micro Henries. (t = 0 for a single layer coil).

Capacitance $q = C\ V$ where q is in Coulombs (ie amps times seconds), C is Farads and V is volts
Capacitance of a parallel plate condenser area A cm^2 and separated d cms and dielectric constant k
$C = 1.11 \times 10^{-6}\ A\ k / (4\ \pi\ d)$ microfarads

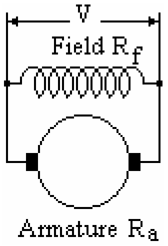
DC Motors and Generators
Motors obey the **left hand rule** and generators the **right hand rule**, (the gener - righter rule).

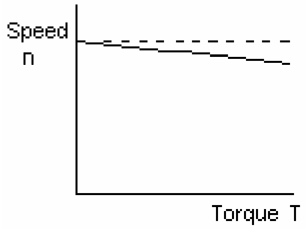




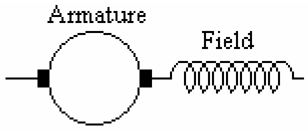
Back emf in DC machine $E = 2p\ Z_S\ \Phi\ rps$ where E is volts, 2p is number of poles, Z_S is number of conductors in series, Φ is in Wb and rps is speed in rev/sec
Power $W = 2p\ Z_S\ \Phi\ I_a\ rps$ where W is watts, I_a is the armature current in amps
Torque $Torque = 2p\ Z_S\ \Phi\ I_a / (2\ \pi)$ Newton metres $= E\ I_a / (2\ \pi\ rps)$ Newton metres
In Imperial units $Torque = 0.117 \times 2p\ Z_S\ \Phi\ I_a$ lb ft $= 0.117\ E\ I_a / (rps)$ lb ft)

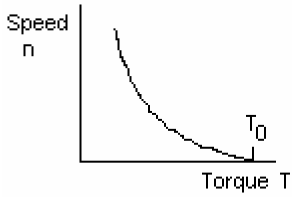
Shunt motor $n = n_0 - m\ T$ where n is speed, n_0 is no load speed, m is approximately constant and T is Torque. $n_0 = V/(2p\ \Phi\ Z_S)$ and $m = 2\ \pi\ R_a / (2p\ \Phi\ Z_S)^2$





Series motor $T = T_0 / (1 + \alpha\ n)^2$ where T_0 and α are approximately constant
$T_0 = 2p\ K\ Z_S\ V^2 / (2\ \pi\ R^2)$ and $\alpha = 2p\ K\ Z_S / R^2$ and $K = \Phi / I = 4\ \pi\ N \times 10^{-7}\ / \Sigma(L / \mu\ A)$



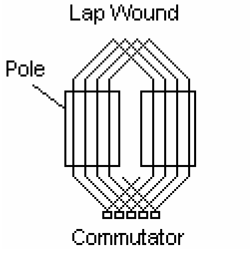


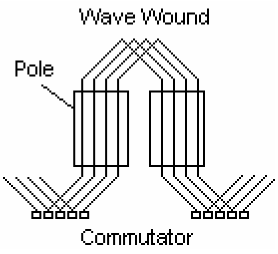
Compound motor has shunt and series windings. This can increase the starting torque for a shunt motor. If wound in opposition, the motor speed can be made nearly constant.

Armature reaction causes a magnetizing force centred between the poles distorting the field and slightly reducing it. **Compensating windings** between the main poles cancel the armature reaction.

Interpoles are small poles carrying armature current between the main poles to improve commutation.

Armature windings can be **lap** or **wave** wound.

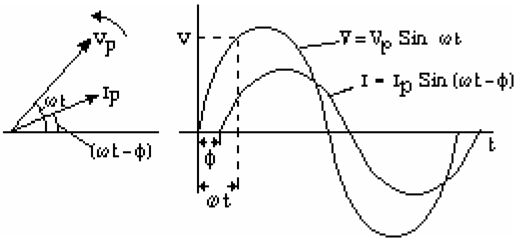




DC shunt generators will fail to excite if there is no residual magnetism or the field resistance is above the critical value for the speed. **DC series or compound generators** require special treatment especially when two or more are in parallel.

Alternating Current AC
AC emf $E = E_p \sin(\omega t) = E_p \sin(2\pi f t)$ where E_p is peak value, f is frequency and t is seconds.
Mean value of E for a half cycle = $2 E_p / \pi = 0.636 E_p$.
Root mean square (rms) value = $E_p / \sqrt{2} = 0.707 E_p$
peak factor = (peak value) / (rms value).
form factor = (rms value) / (average value for ½ cycle)
Square wave peak factor = 1, form factor = 1 Sine wave peak factor = 1.41, form factor = 1.11
Triangular wave peak factor = 1.73, form factor = 1.15

Vector representation of AC voltage and current. The projection on a vertical surface of a vector rotating at constant speed anti clockwise is equal to the value of an AC voltage or current. The phase angle between V and I is the same as the angle between their vectors. The diagram shows the Vector representation of current and voltage where the current lags the voltage This diagram shows the vectors as the peak values. However the rms values are 0.707 times the peak value. Thus the vector diagram shows the rms values to a different scale. Vector diagrams are rms values unless stated otherwise.



Power Factor is Cos φ where φ is the angle between the vectors for V and I
Power in a single phase AC circuit $W = V I \cos \varphi$ watts
Power in a three phase AC circuit $W = \sqrt{3}\, V I \cos \varphi$ watts where V is the voltage <u>between lines</u>

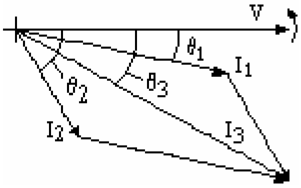
Resistance is higher on AC due to eddy current loss.
$R_f = R_0 [1 + 100 \pi^4 f^2 a^4 / (3 \rho^2)]$ where R_f and R_0 are the AC and DC resistances, f is the frequency, a is the radius of the conductor in metres and ρ is the resistance in microhms / cm cube.
$V = I R$ and the voltage V is in phase with the current I.

Inductance $V = I X_L$ where $X_L = 2 \pi f L$ where L is in Henries. I lags V by π/2. At 50 cps, $X_L = 314 L$

Capacitance $V = I X_C$ where $X_C = 1/(2 \pi f C)$ where C is in farads. I leads V by π/2.
At 50 cps $X_C = 3183/C$ where C is in micro farads.

Inductive Impedance Z = R + jX. $V = I \sqrt{(R^2 + X^2)}$ I lags V by arc tan (X/R)
Capacitive Impedance Z = R + jX. $V = I \sqrt{(R^2 + X^2)}$ I leads V by arc tan (X/R)
Impedance $R_1 + jX_1$ in series with $R_2 + jX_2$ Equivalent impedance = $(R_1 + R_2) + j(X_1 + X_2)$
Impedance $R_1 + jX_1$ in parallel with $R_2 + jX_2$ Put X +ive for inductance, –ive for capacitance
Put $Z_1 = \sqrt{(R_1^2 + X_1^2)}$ and $Z_2 = \sqrt{(R_2^2 + X_2^2)}$ Put $A = R_1/Z_1^2 + R_2/Z_2^2$ and $B = X_1/Z_1^2 + X_2/Z_2^2$
Equivalent impedance is $R = A/(A^2 + B^2)$ and $X = B/(A^2 + B^2)$

Sum of two AC currents.
Add I_1 at phase angle θ_1 to current I_2 at phase angle θ_2 and the result is I_3 at phase angle θ_3
I_3 and θ_3 are obtained by the vector addition of I_1 and I_2.



Star/Delta transformation
Three impedances R + jX in star = three impedances 3R + 3jX in Delta

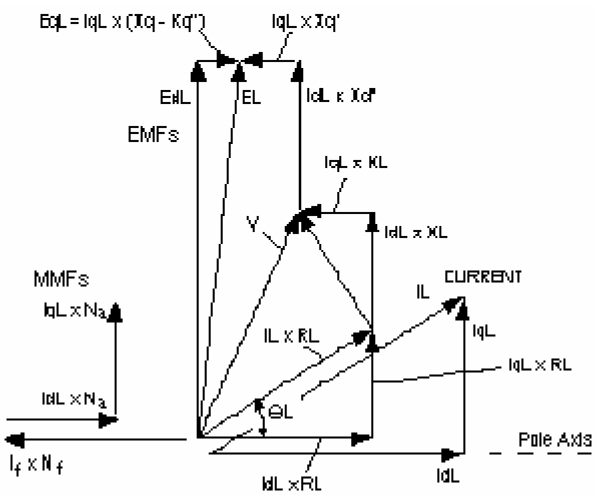
Hysteresis loss Loss = f (area of hysteresis loop) watts/cubic metre with hysteresis loop in Oe and G
Energy in magnetic field Energy = $B^2\ 10^7 / (8 \pi)$ joules per cubic metre
Eddy current loss in laminated core Loss = $\pi^2 f^2 B_M^2 b^2 / (6 \rho)$ watts per cubic metre
Where $B = B_M \sin(2\pi f t)$ is parallel to the lamination, b is the thickness and ρ is the resistivity.

AC generators and motors
Fundamental EMF of generator $E_{RMS} = 4.44\, k_P k_D N f \Phi_{TOTAL}$
where N is (number of turns) / (pairs of poles) and k_P is the pitch factor. If each coil spans an angle of 2λ instead of the full angle π between the poles, then $k_P = \sin(\lambda)$. k_D is the distribution factor due to the phase difference of the emf in each conductor. k_D = (vector sum of emfs) /(scalar sum of emfs)
For n^{th} harmonic, $k_{nP} = \sin(n\lambda)$, and . $k_{nD} = \sin(n\theta/2) / [c \sin(n\theta/2c)]$ where θ = π / (no of phases) and c = slots / phase / pole. Harmonic content can be kept small by suitable values for λ, θ and c.

MMF including harmonics due to a three phase winding in slots
$F = (4/\pi)\, F_{MAX}\, (3/2)\, [k_{1D} \sin(\theta - \omega t) + (k_{5D}/5) \sin(5\theta - \omega t) + (k_{7D}/7) \sin(7\theta - \omega t) + \ldots\ ]$
The third harmonic is blocked by a delta star transformer and can be ignored.

Armature reaction of a current in phase with V gives an mmf between the poles distorting the field.
Armature reaction of lagging currents give an mmf opposing the main field.
Armature reaction of leading currents give an mmf boosting the main field.

Vector Diagram of the emfs, current and mmfs of a synchronous generator.



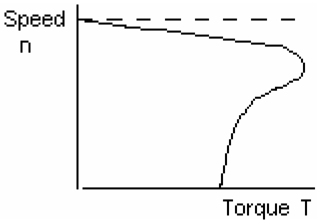
magnitude of EdL
= magnitude of EL

Suffix L signifies on load condition

Automatic Voltage Regulator adjusts the excitation so that at the system design power factor, the voltage is correct whatever the current. If however it adjusts the excitation to give the correct voltage at other power factors, then two machines will not run in parallel. One can supply a huge leading current and the other a huge lagging current. A “droop” is needed to give a lower voltage if the power factor lags by more than the system design. This is achieved by the **compounding**. Faulty Compounding causes unstable sharing of kVAr which can be quite violent.

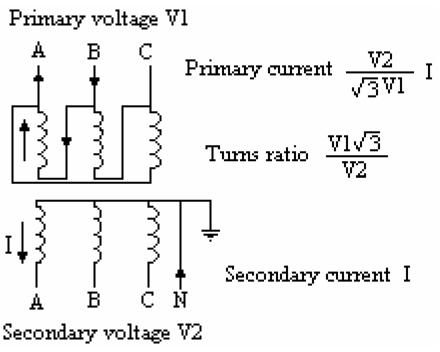
System Faults. When a fault occurs, initially dc currents are induced in the damping winding and main field circuit opposing the demagnetizing effect of the low power factor fault current. These currents die away exponentially causing the fault current to fall. In extreme cases it can fall below the full load value.

Induction motor Power $= 3\, V^2\, (1 - \Sigma)\, R_r\ \ \Sigma / (R_r^2 + X^2\, \Sigma^2)$ watts where the slip $\Sigma = (n_0 - n) / n_0$
Power $= 2\, \pi\, T\, (1 - \Sigma)\, n_0$ watts where T is the torque in Newton metres and n_0 is the synch speed
Torque $= 3\, V^2\, R_r\ \ \Sigma / [\, 2\, \pi\, n_0\, (R_r^2 + X^2\, \Sigma^2)\,]$ Torque is a maximum when $\Sigma = R_r / X$
Put $\Sigma = 1$, Starting Torque $= 3\, V^2\, R_r\ \ / [\, 2\, \pi\, n_0\, (R_r^2 + X^2)]$
If $R_r = X$, the maximum torque occurs when the speed is zero but the motor would be very inefficient. However large motors sometimes have slip rings allowing an external resistance to be added for starting.



The Induction motor speed torque curve. Sometimes there is a kink in the curve at a speed below the speed for maximum torque due to harmonics in the supply. In such cases, the motor may get stuck at this speed , called “crawling”.

Transformers
Power transformers are usually delta primary and star secondary. The primary is supplied through three conductors. Third harmonics are equal and opposite at each end of each primary winding.
Flux $\Phi_{max} = [4\ \pi\ \mu\ A\ N\ I_{max} / L] \times 10^{-7}$ weber
EMF $E_{rms} = 4.44\ N\ \ \Phi_{max}\ \ f$ volts



Delta Star Transformation
Three phase load, primary current equals secondary current times voltage ratio.

A single phase load on the secondary results in a current on two lines in the primary governed by the turns ratio, not the voltage ratio.